Neʻepapa Ka Hana 2.0
Eighth-Grade Mathematics Resources
STEMD² Book Series

TEACHER'S GUIDE

LET'S MAKE DA KINE

STEMD² Research & Development Group
University of Hawaiʻi at Mānoa

STEMD^2 Research & Development Group

Center on Disability Studies

College of Education

University of Hawaiʻi at Mānoa

http://stemd2.com/

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Neʻepapa Ka Hana Eighth-Grade Mathematics Resources

Let's Make Da Kine

Teacher's Guide

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Preface

About the STEMD² Book Series

The STEMD² Book Series for eighth-grade mathematics was developed as part of a technology-enabled pedagogical approach (Ne'epapa Ka Hana model) for teaching mathematics in Hawai'i middle schools. This book series provides Hawai'i middle school teachers resources and training to incorporate problem-based learning, social learning and inclusive pedagogy through a culturally relevant mathematics curriculum. The series currently includes:

Let's Build a Canoe – Student Activities and Teacher's Guide (Common Core aligned)
Let's Play the 'Ukulele – Student Activities and Teacher's Guide (Common Core aligned)
Let's Go Fishing – Student Activities and Teacher's Guide (SBAC aligned)
Let's Make Da Kine / E Hana Kākou – Student Mini Projects and Teacher's Guide in English and 'Ōlelo Hawai'i (Skill development)

The printed and online resources produced by NKH through STEMD² are fully aligned with the Common Core Standards for Mathematical Practice and Content and the Smarter Balanced Assessments for mathematics. Based on the GO Math!® curriculum structure, the NKH STEMD² book series and social learning platform (`community.stemd2.com`) is flexible for teachers to implement partially or fully in their classrooms, as a tool to encourage students' interest and achievement in STEM subjects.

Funded by a three-year grant from the Department of Education's Native Hawaiian Education Act Program, Ne'epapa Ka Hana is a project of the STEMD² Research & Development Group in the Center on Disability Studies at the University of Hawai'i at Mānoa. More information about STEMD² is available online at `www.stemd2.com`.

Unit 1 Activities

Activity 1.1: Map Quest

Introduction:

In ancient Hawaiʻi, when very high ranking chiefs died, special care was taken of their bodies for burial. Their bodies were buried in a shallow hole and covered up. A large fire was then made on top of the body to help separate the flesh from the bones, called *pūholoholo*. The flesh was gathered and put in a container, and the bones were held separately in another container. These containers were then taken by two men who hid them in a secret cave.

One of the men was the keeper, or *kahu*, of the cave. His job was to maintain the secrecy of the location and to take care of the area also. The other person was the *moe puu*, which means 'sleeping together'. He was sacrificed by the kahu because it was believed that the blood would help to protect the cheif's body from evil. This also ensured that no one but the kahu would know the location of the body. In some cases, the burial caves were only accessible by rope along a cliff's face and the kahu would make the sacrifice of the moe puu by simply cutting his rope to have his blood fall below on the rocks.

The reason that great care was taken to keep these burial places secret was that the Hawaiians believed that there were spiritual powers, or *mana*, in the bones of people. *Pūholoholo* had more mana, and creating something out of their bones, like a hook or spear, was believed to be more powerful and more lucky than a normal hook or spear. If the chief's bones fell into the wrong hands, it wasn't only possibly helping out the warring groups, but it was also a great disrespect to the family of the pūholoholo.

In this activity we will use a map to find a hidden treasure. Note that in this activity we will *not* use traditional Native Hawaiian measurements, which make use of your body to measure things.

Materials needed:

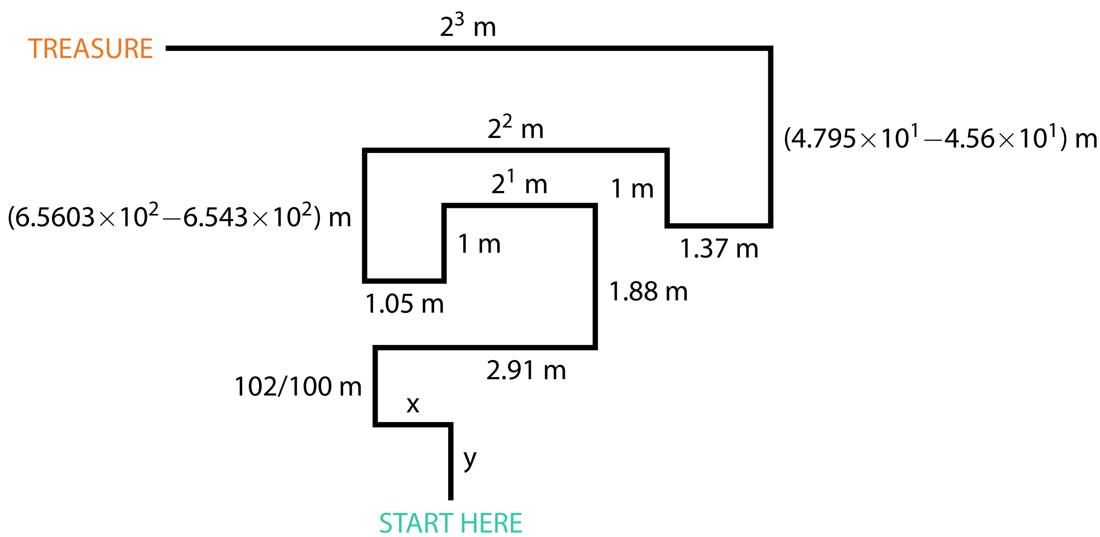
1. Measuring tape
2. String
3. Some field area
4. A treasure
5. A few decoy items

Setup instructions:

1. Determine where your starting point is.
2. From the starting point walk at least 4 meters left and write down this distance.
   LEFT = ___________
3. From the previous point walk at least 6 meters up and write down this distance.
   UP = ___________
4. Solve for $x =$ LEFT -2.77m and $y =$ UP -4.95m. Provide these lengths at the start of this activity. (They're missing on the map.)
   $x =$ ___________
   $y =$ ___________
5. Place the treasure here.
6. Now place decoy items around the field area.

Activity

Keoni has found a map that he believes will lead to a secret buried treasure. He knows where the starting point is and which direction the map faces. Can you help him to find the treasure?



Keoni's treasure map.

Instructions:

1. Using what you know about scientific notation, rational numbers, and exponents, find the distance of each leg of the journey.
2. Using a piece of string, measure out enough for each leg of the journey and mark the distance in increments by placing a piece of tape at the appropriate position on the string.
3. As a group, start from the indicated position and direction. Use the string as a guide, map out the path and find the treasure.

Questions:

1. Why would someone create such a crazy map?

2. If you wanted to help a friend find the location of this treasure, what instructions would you write for him/her?

These back and forth steps may be summarized in several ways. For example, students may write "From the starting position, walk $x + 2.77$m left and then $y + 4.95$m up."

3. What was the treasure? Share some photos in the online gallery.

Unit 2 Activities

Activity 2.1: Hawaiian Trading Company

Introduction:

Traditionally, Hawaiians used a bartering system to exchange goods and services. In modern times, we have currency that acts as a standard way of paying for items, but bartering still exists even today. Students trade cards, pogs, games, and even items within video games. This idea of bartering and how we value something helps us make good decisions and trades.

Materials needed:

1. Index cards (4-5 per student or group)
2. Pencils, crayons, or pens

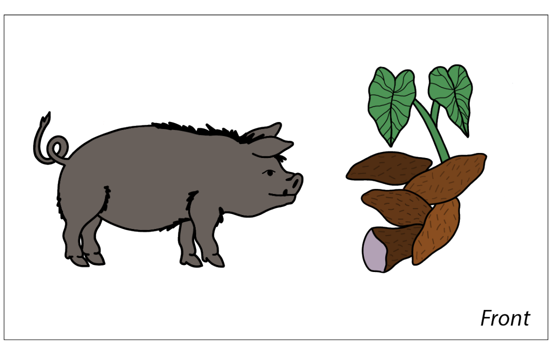
Activity:

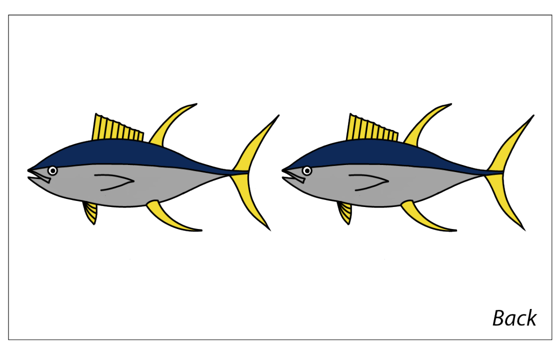
Imagine you lived in ancient Hawaiʻi, and you wanted to throw a lūʻau for your friends and family. Your family lives by the ocean and are excellent fishermen, but you would like some pigs for your guests too, so you decide to visit a great hunter. The hunter says, I'll give you 1 pig and 5 kalo for 2 of your fish. You decide to take the deal and keep track of the deal on a piece of tapa.

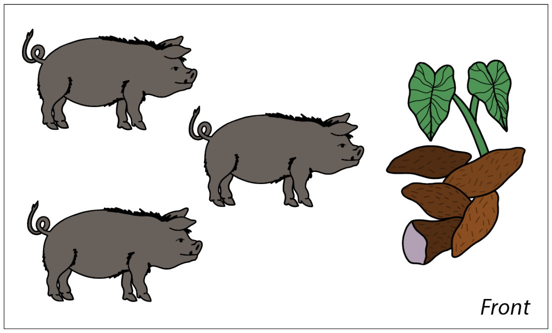
When you arrive back home to gather your fish to trade with the hunter, you find out that there are more guests coming than you had anticipated. You now need more pigs. You visit the hunter again, and he offers to give you 3 pigs and 5 kalo for 6 of your fish. You keep track of this on your tapa as well. This is a great deal, so you head home again to gather your fish for the trade.

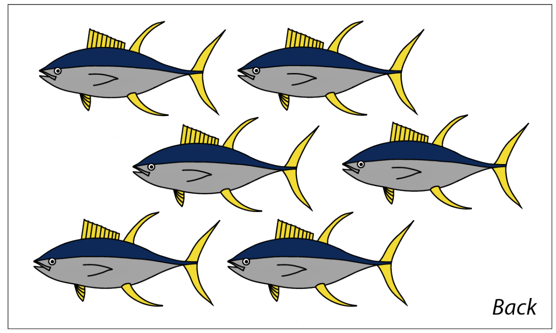
When you get home, your mother tells you that there are *even more* people attending than you planned, so you head back to the hunter who tells you that he can give you 7 pigs and 5 kalo for 14 fish. After keeping track of this new offer, you think you finally got this figured out and head back to home. However, come to find out, now you only need 4 pigs due to some cancellations.

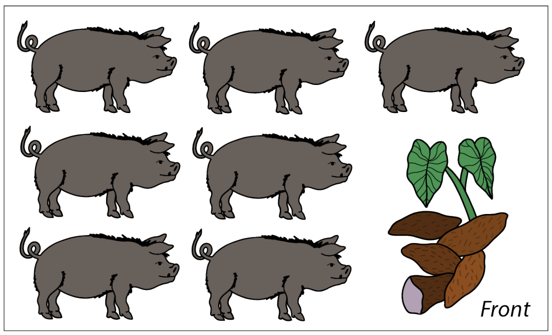
Instead of heading back to talk to the hunter, or hauling extra fish around, you decide to just bring the amount of fish you need for the trade. Let's summarize the information that we have so far and present them on index cards.

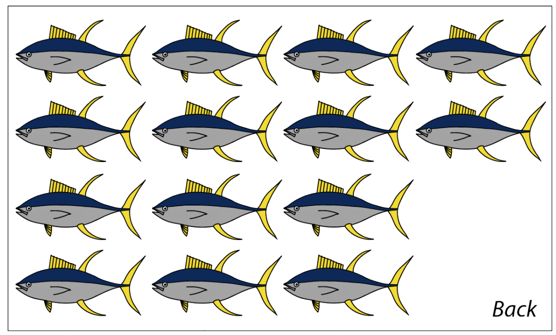












From these cards, we can see that each pig is worth 2 fish, since there is always a double amount of fish per each pig. Therefore, if you want 4 pigs, you would need to give the hunter 8 fish. You will also get a bonus of 5 kalo too!

Now come up with your own scenarios and make your own cards for your classmates to try and figure out how you value things and what's a fair trade for you. It doesn't have to be pigs and fish, but can be anything you are interested in or would trade. Try to see if you can figure out how your classmates value things, and who would be best to barter with if you had to trade some items.

Activity 2.2: Fishing Poles and Fishing Spots

Introduction:

Many Hawaiians have a deeply personal and spiritual relationship with the rivers and the seas, and feel a kuleana, or a *responsibility*, to take care of the water because it takes care of them every single day. In fact, most of the meat in the traditional Hawaiian diet was seafood. Hawaiians were also skilled in making and using hooks and lines, lures, nets, basket traps, and spears to catch fish. In this activity, we will make a quick emergency fishing pole.

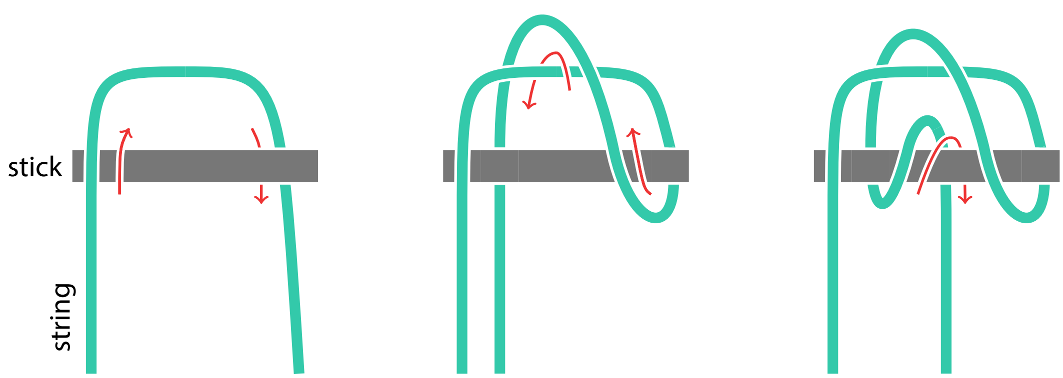
Materials needed:

1. A stick or branch, about 4 or 5 feet long
2. A string, line, or thread about 12 feet long
3. Duct tape or masking tape

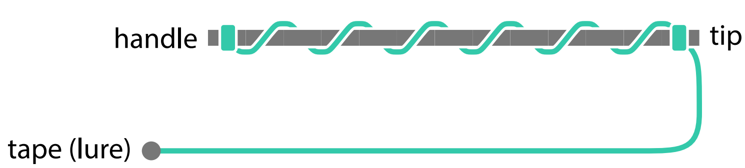
> This material list is for *one* student. Please scale accordingly. Bamboo is a good choice for a stick and usually obtainable in craft stores.

Activity:

First use the following knot to tie the *end* of your string to the *handle* of the stick.



Next, wind the string down the length of the stick until you have about 4 or 5 feet of string remaining at the tip. Tie the *tip* with the same type of knot (above). The winding string should be tight on the fishing pole. If it is not tight enough, then twist the two knots until the string tightens, and tape the knots down to keep them from untwisting.



Finally, wrap some tape around the end of the string to make a lure and to give the string a little bit more mass.

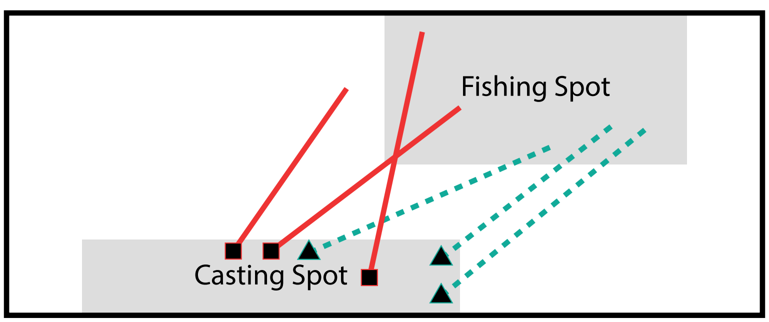
A skilled fisherman is able to cast their line into a fishing spot without issues. This means that the line does not accidentally break or get stuck. This also means that their line doesn't get tangled with the lines of other fishers.

In this activity, we'll first divide the students into two or more teams. Then the teacher will mark a part of the floor as the **Fishing Spot** and another part as the **Casting Spot**.

Everyone needs to cast their fishing line from inside the Casting Spot. When the teacher says to start, all students have to cast their fishing line towards the Fishing Spot. After **five seconds**, everyone must stop *immediately* and put their fishing poles down. A team gets one point whenever a fisherman on that team:

1. gets their lure into the Fishing Spot **and**
2. their line does not intersect anyone else's line.

Here's an example!



In the image above, we see that the ▲ team gets 2 points for this round and the ■ team gets 0 points.

Play a few rounds to see which team can get the most points!

Feel free to change some details to adjust for your classroom dynamics. For example, you may allow more than five seconds to cast a fishing pole to accommodate students with disabilities.

You may also make the Fishing Spot smaller or larger and the Casting Spot closer or further away to adjust the difficulty of this activity.

Unit 3 Activities

Activity 3.1: Battle at the Beach

Introduction:

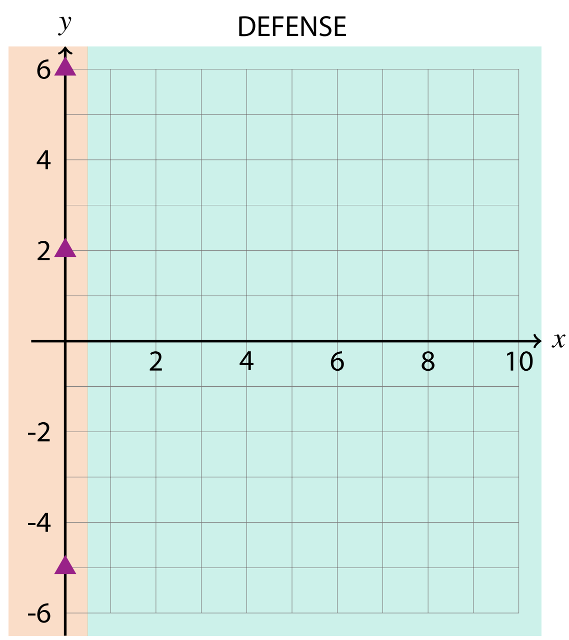
King Kamehameha I (a.k.a Kamehameha the Great) was the first person to unify all of the Hawaiian Islands into a kingdom. To unite a chain of islands, a leader must be skilled at bringing a fleet of war canoes onto an enemy's beach while keeping the enemies' armies away. In this activity, we will use linear equations to defend our beach against our invading classmates.

Materials needed:

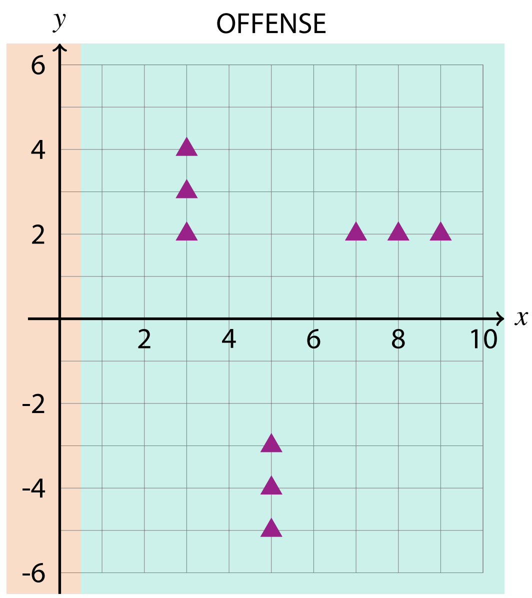
1. A pen/pencil
2. Scratch paper

Activity:

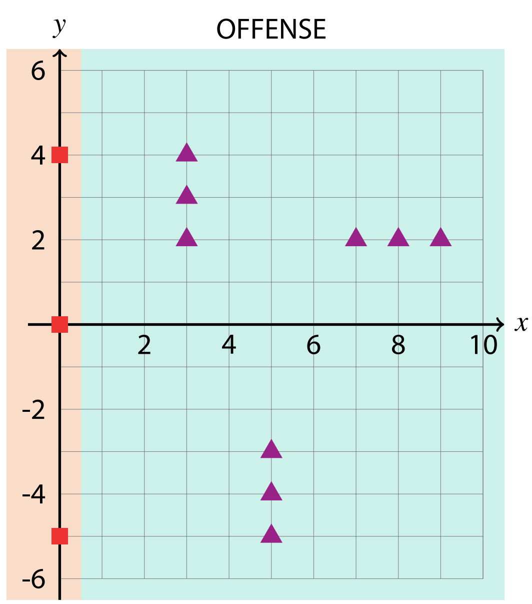
1. Find an opponent and print out two copies of the game board, one for you and one for your opponent. You'll find the game board at the end of these instructions.
2. Sit somewhere so that you cannot see each other's game board. Maybe you need to stand up a big book between you and your opponent.
3. The DEFENSE side shows your beach, with your guards and an ocean that you must defend.
4. On the DEFENSE side, choose three positions along the y-axis to place your guards. Mark these positions with a triangle. Note: only use integer values for your locations (for example, (0, 2) and (0, -3) are okay but (0, 2.5) is not okay).
   Example:



5. The OFFENSE side shows your opponent's beach where you must send your war canoes.
6. On the OFFENSE side, decide where you'll place your three war canoes. Each war canoe carries three warriors so you need to place them three-in-a-row or three-in-a-column. Mark the positions of your warriors with a triangle. Note: like before, only use integer values.
   Example:



7. Give your guard locations to your opponent and get their guard locations as well. Mark your opponent's guard locations on your OFFENSE map with a square.
   Example: If your opponent had guards at $y =$ -5, 0, and 4 then you'd end up with this.



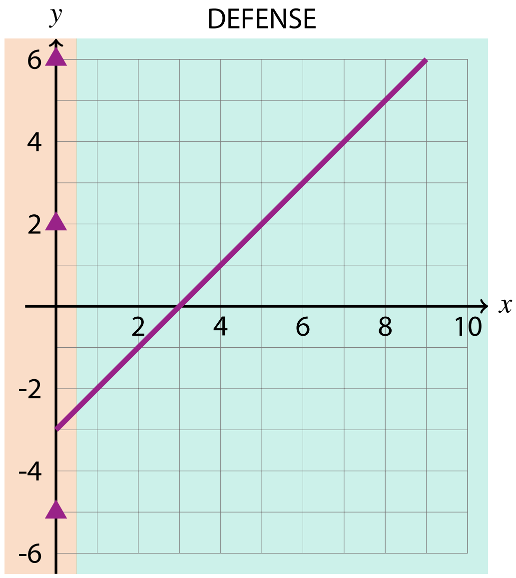
8. Use Janken Pon (rock paper scissors) to determine who goes first. Take turns defending your beach by "throwing spears" into the ocean at the attacking warriors.
9. **The first person to get rid of all of their opponent's warriors wins!**

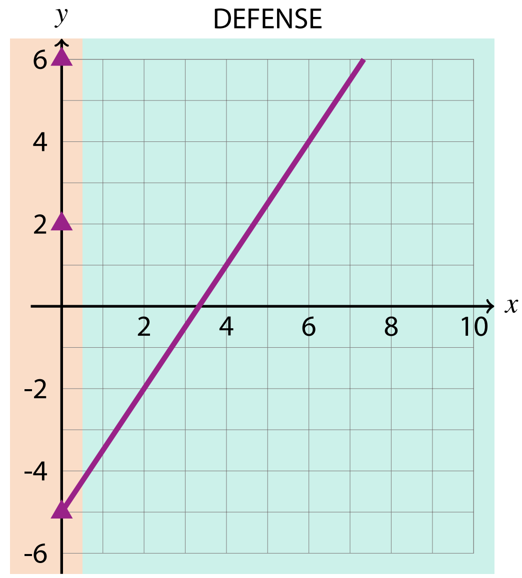
When it is your turn to throw a spear:

1. Give a linear equation that intersects one of your guards. This is where the spear will fly.
   Example:

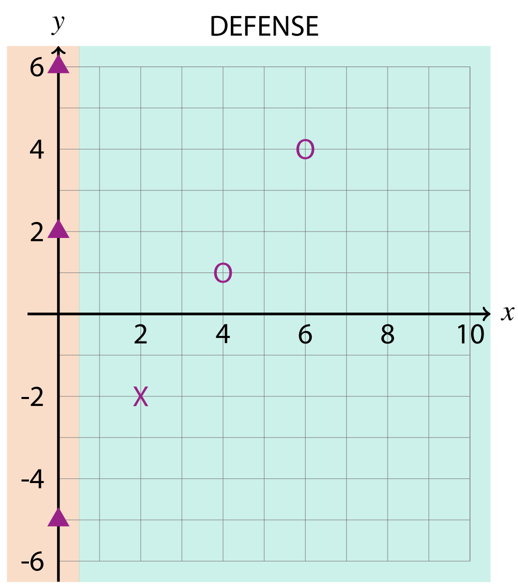
   $y = -3 + x$ is not an okay equation because the line does not intersect a guard so a guard cannot throw this spear.

   $y = -5 + \frac{3}{2}x$ is an okay equation because it is linear and intersects a guard so a guard can throw this spear.





2. Ask if there are any warriors at the places where the spear intersect the grid. Mark an X on your DEFENSE if there are, and mark an O if there aren't.
   Example: $y = -5 + \frac{3}{2}x$ only intersects the grid at (2, -2), (4, 1), and (6, 4). If your opponent tells you that you hit a warrior at (2, -2), but missed at (4, 1) and (6, 4), then you make these marks.

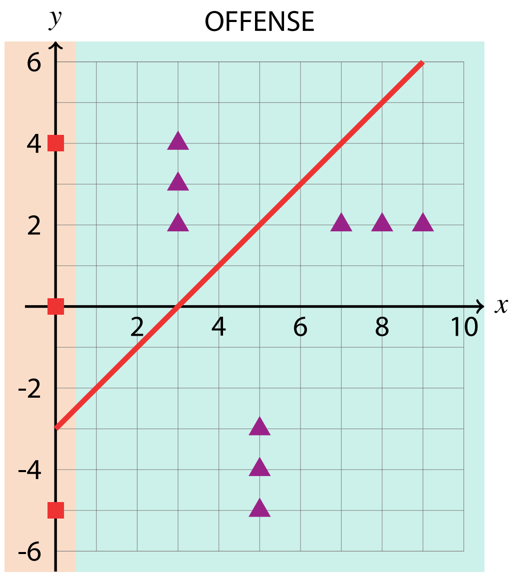


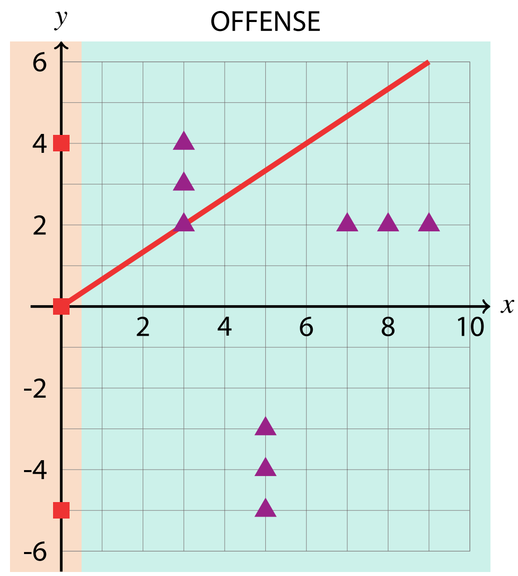
When it is your opponent's turn to throw a spear:

1. First check if their throw is possible. If it is not, then tell them that they lose their turn. *Example:*

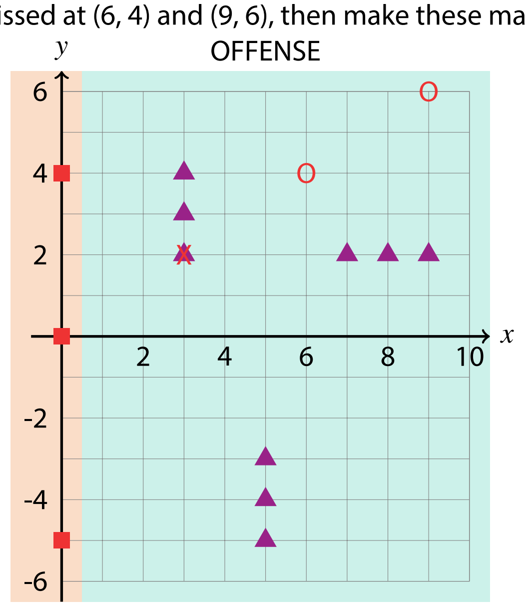
   $y = -3 + x$ is not an okay equation because the line does not intersect a guard so a guard cannot throw this spear. So your opponent loses a turn.

   $y = \frac{2}{3}x$ is an okay equation because the it is linear and intersects a guard so a guard can throw this spear. So your opponent's turn continues.

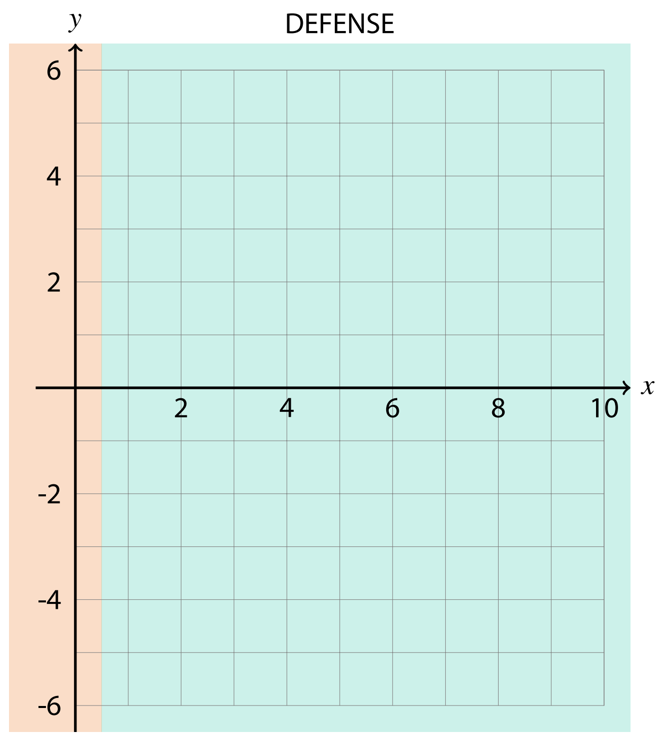


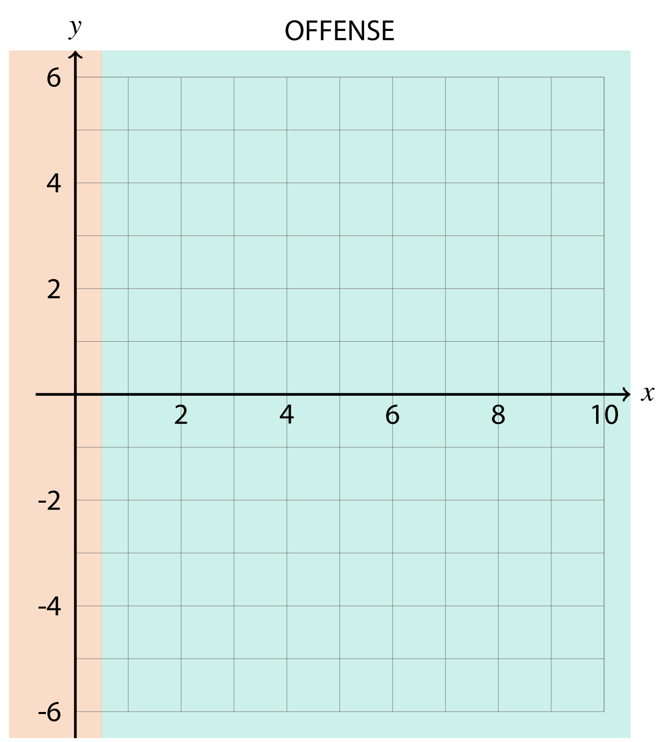


2. Let your opponent know if there are any warriors at the places where the spear intersect the grid. Mark and X on your OFFENSE if there are and mark and O if there aren't. *Example:* $y = \frac{2}{3}x$ only intersects the grid at (3, 2), (6, 4), and (9, 6). Tell your opponent that they hit a warrior at (3, 2), but missed at (6, 4) and (9, 6), then make these marks.



Battle at the Beach: game board





Unit 4 Activities

Activity 4.1: A Royal Pattern

Introduction:

Koa decides that he wants a new cape with his special pattern. In this activity, we'll see how he can dilate a small picture of his drawing to decorate his cape.

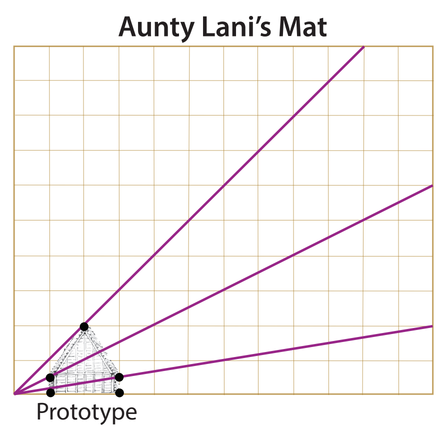
Materials needed:

1. Permanent marker
2. Rulers
3. Brown paper bags (for a cape)
4. Tape

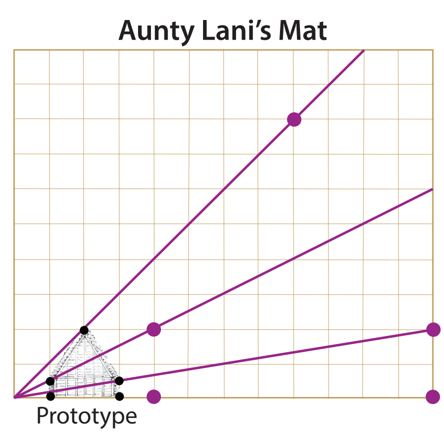
Activity:

He doesn't really know what he's doing so he goes to his aunt for help. Aunty Lani is an expert in these kind of things so Koa visits her and watches her dilate a picture of a house on her mat.

1. Aunty Lani chooses important points on the prototype. Then she uses strings to draw lines from the origin (bottom left corner) through the points.

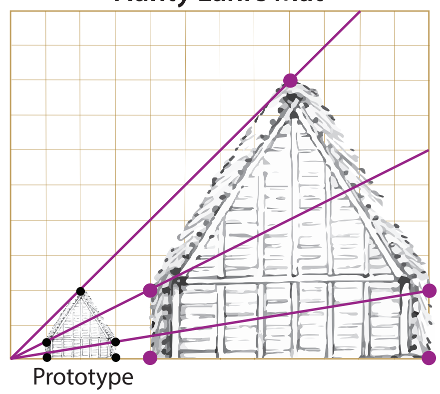


2. Aunty Lani moves the points down the lines from Step 1, making sure that they all line up correctly. *Pay attention to which dots line up vertically and which line horizontally.*

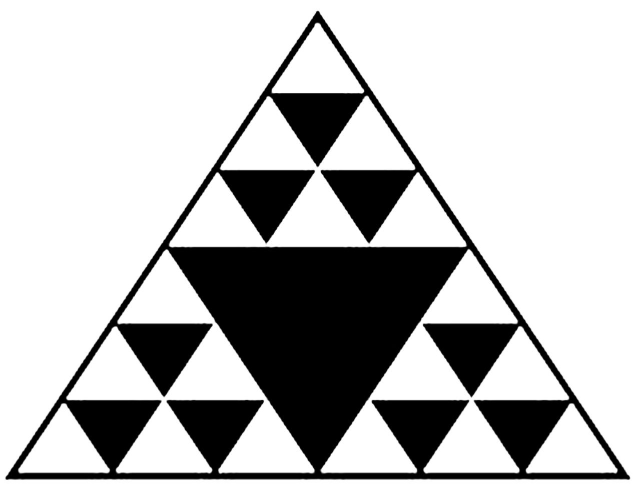


3. Now that Aunty Lani knows where all the special points end up, she can easily fill in the rest of the missing pieces.

Aunty Lani's Mat



Let's use the same principles demonstrated by Aunty Lani to dilate Koa's cape pattern. Koa wants this pattern 3 times bigger than the picture given below printed onto his cape.



Cut open the paper bags to get an adequate surface for drawing and tape together enough to make a cape. Place the small logo to the left of the paper bag cape and use dilation to enlarge it. Try to get it to be 3 times larger than the original.

Activity 4.2: Poi Dance

Introduction:

In New Zealand, many Maori people practice a traditional art form called *poi*. Poi is a beautiful dance that includes a spinning ball on a string in different patterns. Sometimes sticks are used in the dance too.

Materials needed:

1. Thick string (like yarn)
2. Scissors
3. Paper clip
4. Lots of scratch paper or newspaper
5. Masking tape
6. Permanent markers

The string on the poi sphere is typically the length of a hand or the length of an arm. The poi sphere can sting if it is swung at another student. If this is a concern in your classroom, then please give the students a short string. This significantly reduces its energy when swung.

This activity does not work well with other types of tape. Most tapes, besides masking tape, tend to be too rigid and difficult to wrap, or too glossy and difficult to draw on.

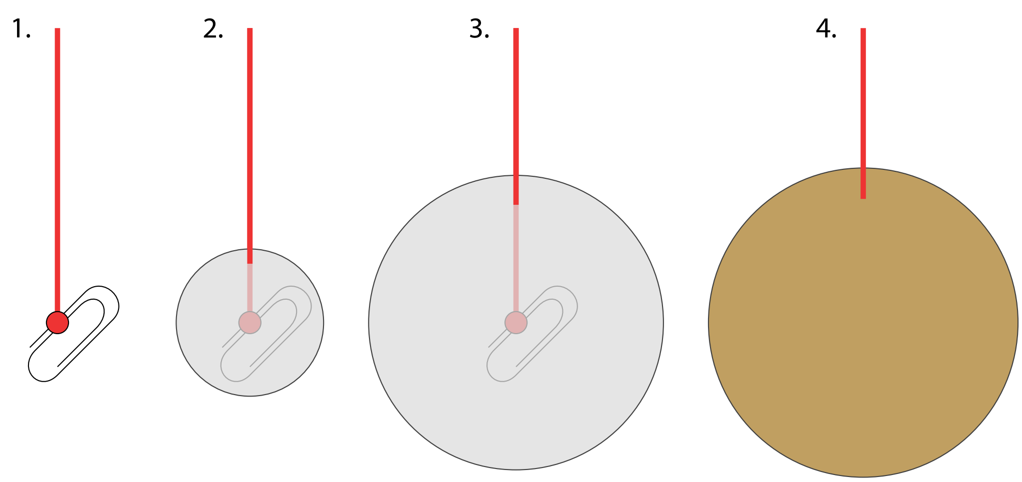
Activity

1. First we need to find a way to connect our poi spheres to the string. We'll do this by tying a big strong knot onto the paper clip.

2. Then we'll start building our poi sphere by tearing small pieces of paper and crumpling the paper around the paper clip.

3. As the poi sphere gets larger, we can wrap larger pieces of paper. Let's wrap until the poi sphere is about as big as two fists held together.

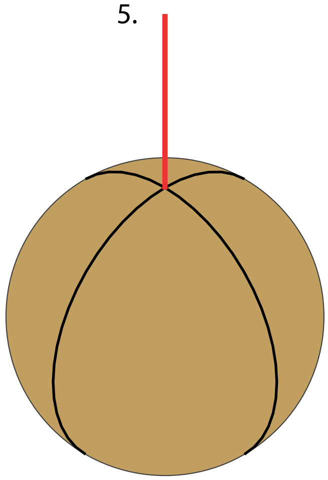
4. When the poi sphere is big enough, we'll cover it with a thin layer of masking tape.



5. Draw two rings on the poi sphere. Make sure that the rings intersect at the string and at the point opposite to the string. This should split your poi sphere into four sections like cutting an orange into four wedges.

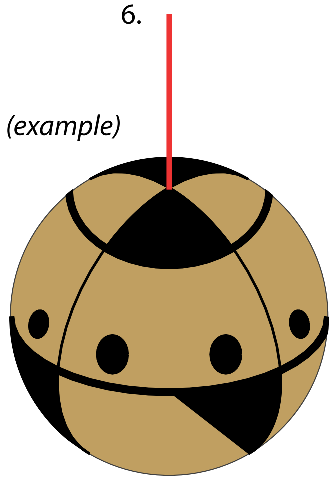
6. Using the rings as guidelines, draw your own designs with markers! Draw lines and shapes and transform your drawings to make patterns. Try to use translations, rotations, reflections, and dilations if you can.

5.



6.

(example)



Unit 5 Activities

Activity 5.1: Proof of the Pythagorean Theorem

Introduction:

Building materials can be hard to come by in the remote islands in the Pacific. Because of this, Pacific Islanders became really good at making the most of what they have. For example, people in Micronesia traditionally built larger canoes by stitching and tying smaller pieces of wood together. Also, Hawaiians weaved small leaves together to make huge floor mats and canoe sails. In this activity, we will learn how to make a big square using any two small squares, and we'll do this without wasting any material. We'll also learn more about the Pythagorean Theorem.

Materials needed:

1. 2 sheets of paper, square shaped, different sizes
2. Pencil or pen
3. Ruler or straight edge
4. Tape
5. Scissors

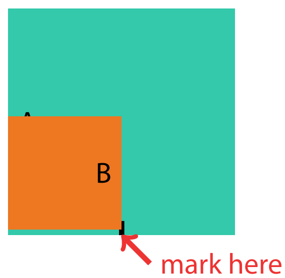
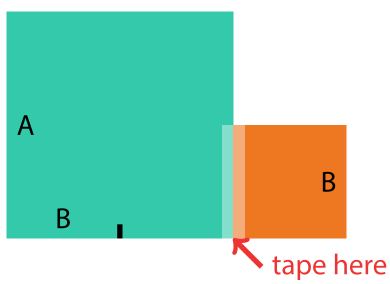
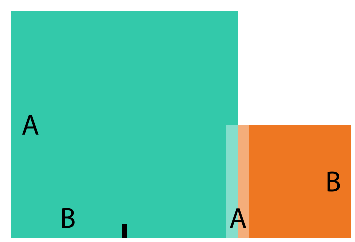
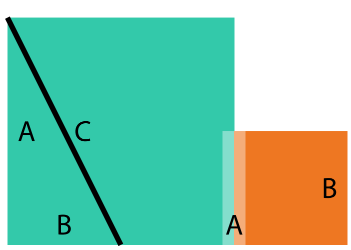
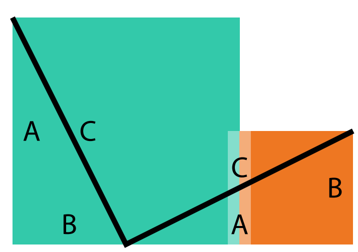
The challenge of this activity is to take two squares of any size, cut the pieces up, and tape it all back together to form one larger square. The area of the final square must be equal to the sum of the original squares, meaning that there are no gaps, no overlapping pieces, and no unused paper. We recommend that the students first try the challenge on their own without looking at the solution below.

Activity

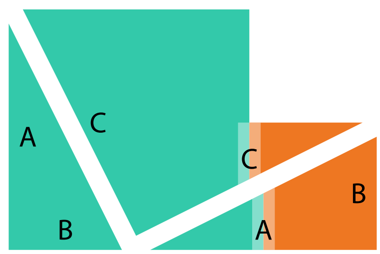
The Pythagorean Theorem tells us if a right triangle has sides of length A, B, and C, where C is the length of the hypotenuse, then $A^2 + B^2 = C^2$. In this activity we will prove it.

1. First we will make two squares. One will have sides of length A and area A^2, and the other will have sides of length B and area B^2.
2. Then we will make a right triangle with sides of length A, B, and C, where C is the length of the hypotenuse.
3. Next we will cut the two squares and tape them back together to make one big square with area C^2.

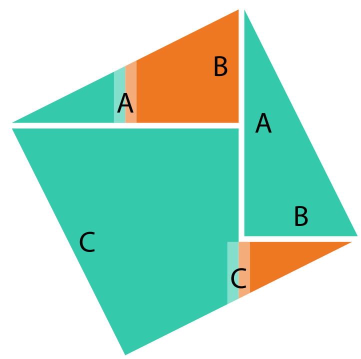
Step 1: Cut out two squares. They don't have to be different sizes but this activity is more interesting if they are.	**Step 2:** Label the left side of the bigger piece A and the right side of the smaller piece B.
	

Step 3: Use the smaller square to make a mark on the bottom of the bigger square that is B away from the left side.	**Step 4:** Label this distance B.
	
Step 5: Tape the left side of the smaller square to the bottom of the right side of the bigger square.	**Step 6:** Label the distance between the marker and the right side of the small square A. Talk to your friends to make sure everyone knows why this distance is A.
	
Step 7: The edges A and B on the left side are perpendicular so we can use the ruler to draw a hypotenuse C.	**Step 8:** The edges A and B on the left side are also perpendicular so we can use the ruler to draw another hypotenuse C. Talk to your friends to make sure everyone knows why this C is the same as the last one.
	

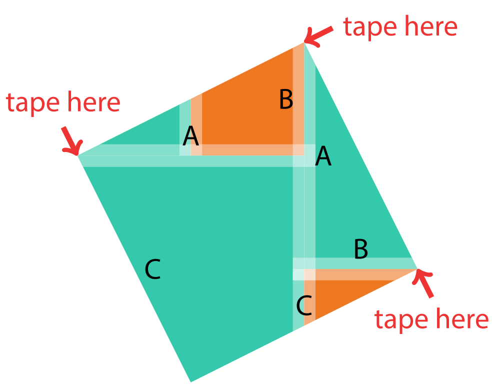
Step 9: Now let's cut along the hypotenuses to get two right triangles.



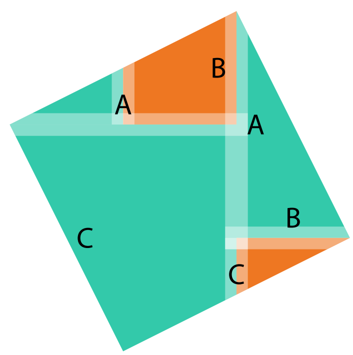
Step 10: Use translations, rotations, and/or reflections to move the right triangles to make a large square. There are many ways to do this.



Step 11: Tape the pieces down.



Step 12: We're done! You started with two squares of areas A^2 and B^2.



What is the area of this final square? How is this area related to the areas of the original squares?

Activity 5.2: Pythagorean ʻUpena (Net)

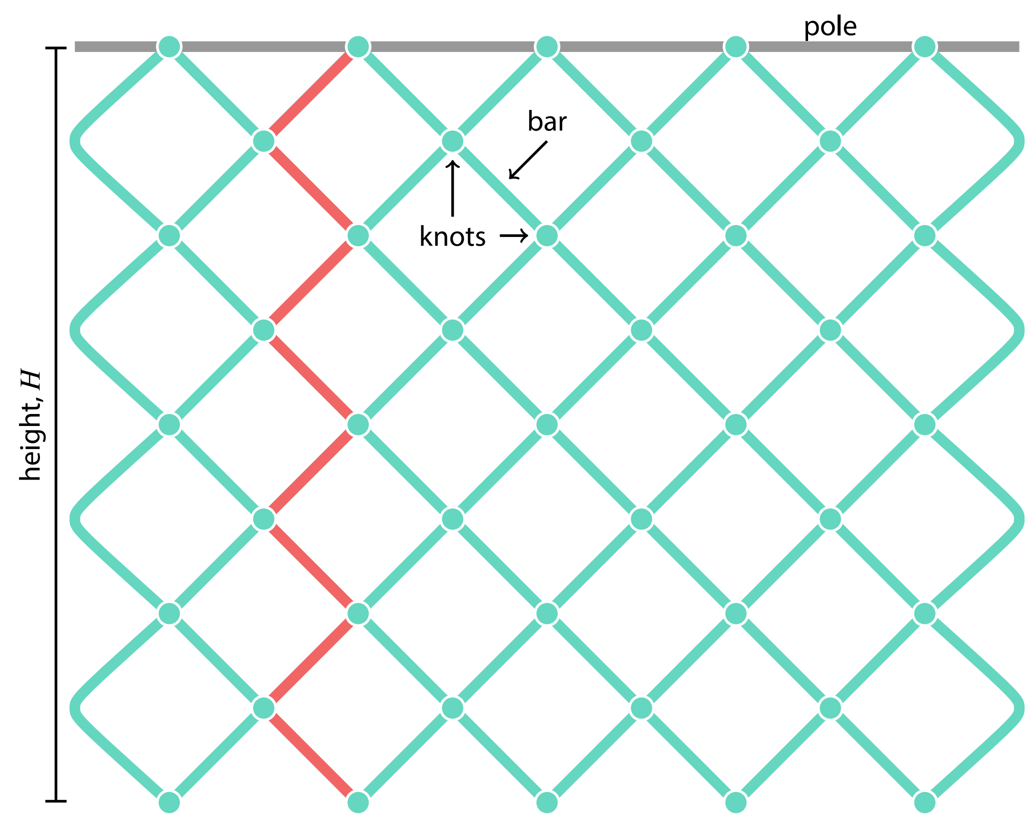
Materials needed:

1. String
2. Scissors
3. Ruler
4. Plastic pole or a long smooth stick

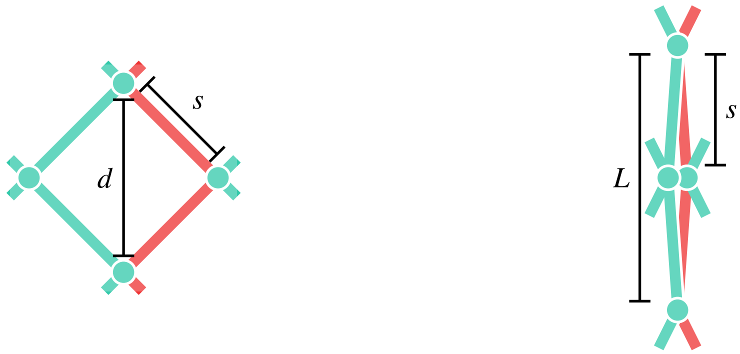
Activity

Kuʻulei is making a net. She knows how tall she wants her net to be and how big each diamond-shaped hole should be. She doesn't want to waste any string or make the knots uneven. With a neighbor or a group, let's use our knowledge of triangles and irrational numbers to help her plan out her net design.

In the picture below, we see that the net is hung from a pole. This makes it easier to sew/tie the net together. The net is made up of strings with their centers tied to the pole. *Half* of a single string is shown in the diagram in red. The net is made up of **knots** and **bars**. All bars are of equal length, s, and the height of the net is H.



Even when you know the length of the bars, calculating the height can be harder than you think. When the net is light and empty, the holes are wide open and the height of each hole is the diagonal d. When the net has a lot of weights on the bottom and is full of fish, then two corners of each hole will stretch until the hole closes. The height of the hole then becomes what's called the "eye length", L.



1. The following equations are true. Talk to your neighbor and explain why.

$$\text{Light net:} \quad d = \sqrt{2} \times s$$
$$\text{Heavy net:} \quad L = 2 \times s$$

You can write down the explanation here if you want. It'll help make the rest of this activity easier.

In the light net, the two sides of length s and the diagonal of length d form a right triangle with the diagonal as the hypotenuse. The Pythagorean Theorem gives us the first equation. In the heavy net, the corners of the hole are stretched until the hole closes. Then we have a line made up of two sides of length s, which is our second equation.

2. So the height of each hole is at least d and at most L. If we have n holes from the pole straight down to the bottom of the net, then our height is at least $d \times n$, and at most $L \times n$.

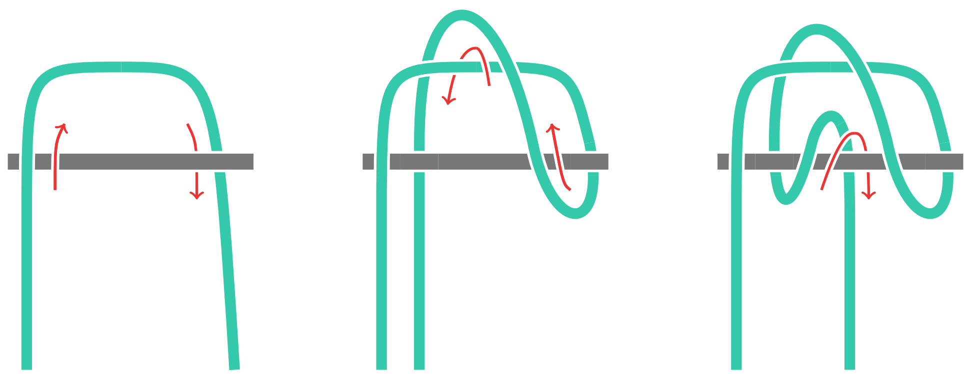
Talk to your neighbor and explain why the following inequalities are true.

$$H \geq \sqrt{2} \times s \times n$$
$$H \leq 2 \times s \times n$$

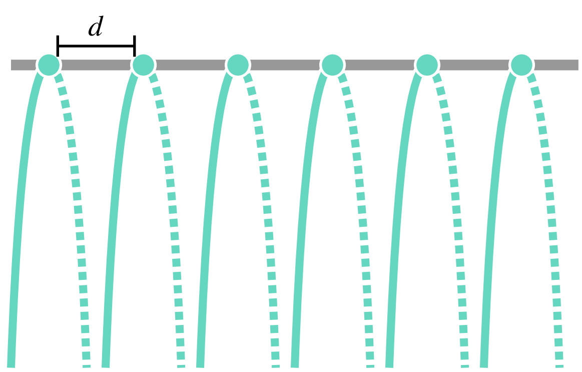
3. Kuʻulei wants a net that is 5 feet in height. She also wants the sides of the holes to be 3 inches. What is an okay number of holes n from top to bottom? (There are more than one possible answer).

In inches, have that $H = 60$ and $s = 3$. So the inequalities give us $10 \leq H \leq 14.142$. Any integer from 10 to 14 will make good choices for n.

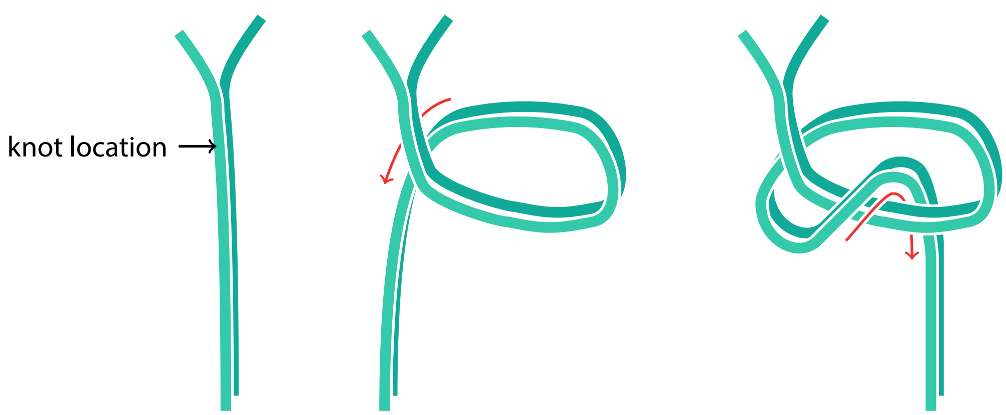
4. Now that we've helped Kuʻulei design her net, let's use what we learned to make our own net. To begin, let's get a smooth stick or pole and cut some pieces of string to tie to it. We'll leave the calculations up to you to decide how long each piece should be. Once you choose values for H and s, then you can calculate everything else. The following diagram shows us how to tie each string to the pole.



Please tie this knot in the center of each string and space them a distance of d apart on the pole. Remember that d is the length of the diagonal of the holes you want.

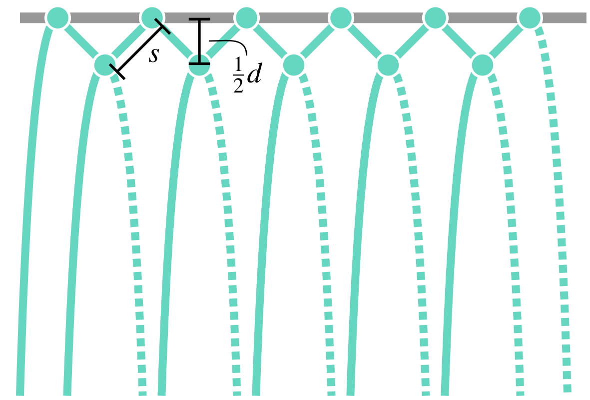


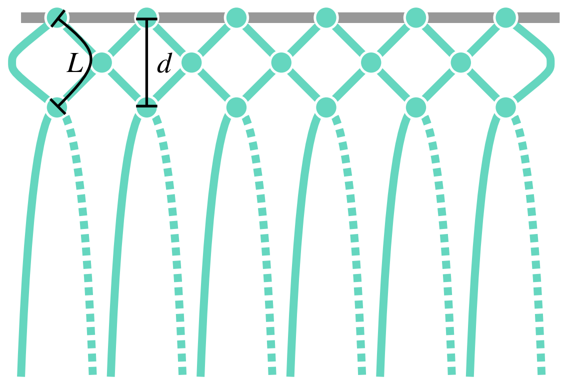
Now we can grab pairs of strings and knot them together to make the net! Here's how we knot two neighboring strings together.

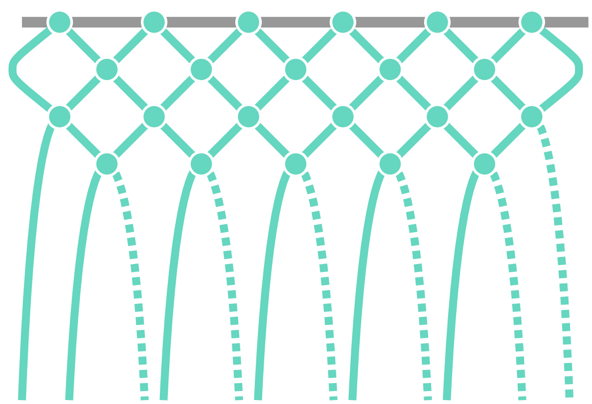


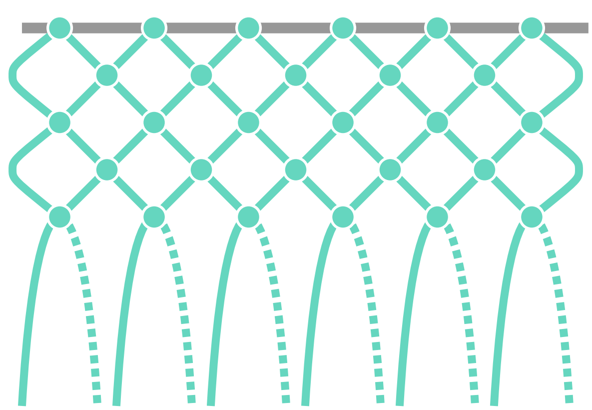
Continue to tie the strings one row at a time. Make sure to take careful measurements as you go so that the net will come together nicely. It's up to you whether you want to measure the holes or the strings!

When you are pau (done) with this activity, slide the knots off of the pole to get your completed net!









Unit 6 Activities

Activity 6.1: 3D Printed Canoe

Materials needed:

1. 3D Printer

Activity

We recommend trying this activity near the end of the school year. We're going to use a little bit of everything that you've learned this year. It's best to do this activity with a group of your classmates, so that you can talk about each question and answer together. The questions can get pretty messy, but it is much better to work together and not get all the answers than it is work alone without talking.

> For schools that are partners with Neʻepapa Ka Hana, please contact us for assistance with 3D printing needs.

Introduction and note on units

In this activity we will design a canoe that is the right size for six paddlers. Then we will 3D print a small model of the canoe.

Note that we will be using metric units for this activity. The numbers and units actually come out a lot nicer with the Metric System as opposed to the U.S. Customary System. This is one reason why most countries around the world use the metric system, as well as all scientific research.

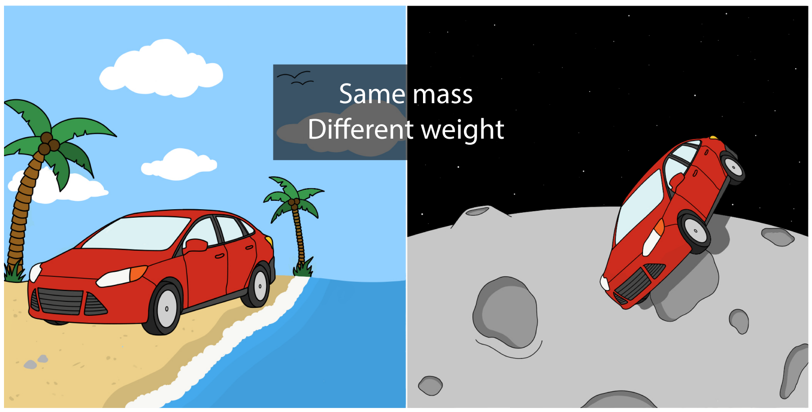
Measurement	Metric unit	US Customary unit
Length	Meter (m)	Feet (ft)
Speed	Meters per second (m/s)	Feet per second (ft/s)
Acceleration	Meters per second squared (m/s^2)	Feet per second squared (ft/s^2)
Mass	Kilogram (kg)	Pound (lb) – sometimes
Weight	Newton (N)	Pound (lb) – sometimes

Part 1: Weight of the Canoe

Here's a table of the six paddlers and their masses.

Name	Mass
Keola	90 kg
Jim	92 kg
Lehua	76 kg
Jaime	67 kg
John	80 kg
Kapono	78 kg

In U.S. Customary units, mass and weight are often talked about in the same way. But in reality, they're actually very different. Mass measures "how much stuff is in something" and weight measures "how hard stuff pushes down." For example, cars are made up of a lot of stuff like metal, rubber, and gasoline, so it has a lot of mass. It is very difficult to pick up a car, so it also has a lot of weight. Imagine the same car on the moon. This car is still made up of all the same stuff, but it is now very light since there is little gravity. So on the moon, the mass is the same, but the weight is almost gone. For most objects, its *weight may come and go*, but its *mass stays the same*.



Force is mass times acceleration and measured in Newton(s). 1 kg $\times$ m/s^2 = 1 Newton. If the acceleration comes from gravity, then the force is called "weight."

To find the weight of the canoe, let's say that the acceleration due to gravity is g m/s^2. You actually don't need to know the value of g in this activity so we won't tell you, but you can look it up on the internet if you want. We have that **weight** = **mass** $\times$ **g**.

Let's assume that the mass of the empty canoe is zero. What is the total **weight** of the canoe with this team sitting inside?
Hint: You should still have g somewhere in your answer.

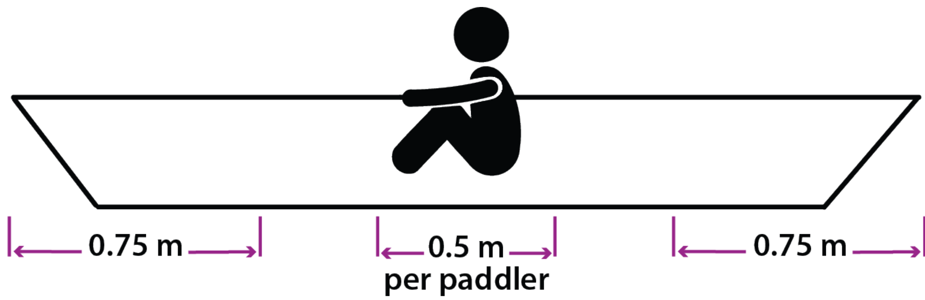
$$\begin{aligned} \text{total mass} &= 90 \text{ kg} + 92 \text{ kg} + 76 \text{ kg} + 67 \text{ kg} + 80 \text{ kg} + 78 \text{ kg} \\ &= 483 \text{ kg} \\ \text{total weight} &= 483 \text{ kg} \times g \ \tfrac{\text{m}}{\text{s}^2} \\ &= (483 \times g) \text{ N} \end{aligned}$$

Note: Later in this activity, the g cancels out which is why we never need its value. However, some students will be distracted by it and struggle to complete the following parts. For those students, we recommend giving the value $g \approx 9.8\text{m/s}^2$. If you're using this value for g then the weight is 4733.4 N. Using 9.8 m/s^2 instead of g will simplify the conceptual challenges of the activity at the cost of introducing a little bit more arithmetic.

Part 2: Minimum canoe size

Now we need to figure out the smallest size that our canoe needs to be in order to fit six paddlers.

a. Suppose that no one sits at the ends of the canoe and each end is 0.75 meters long. Suppose that everyone is seated in the canoe in a line and each person needs at least 0.5 meters length-wise to sit comfortably. If p is the number of paddlers on the canoe, write an equation for L, the minimum length of the canoe.



$L = 0.5p + 1.5$

b. We want to design a canoe for six paddlers, what is the minimum length of our canoe? (Use your equation from Part a.)

$$\begin{aligned} L &= 0.5p + 1.5 \\ &= 0.5(6) + 1.5 \\ &= 3 + 1.5 \\ &= 4.5 \end{aligned}$$

A canoe for 6 paddlers should be at least 4.5 m long.

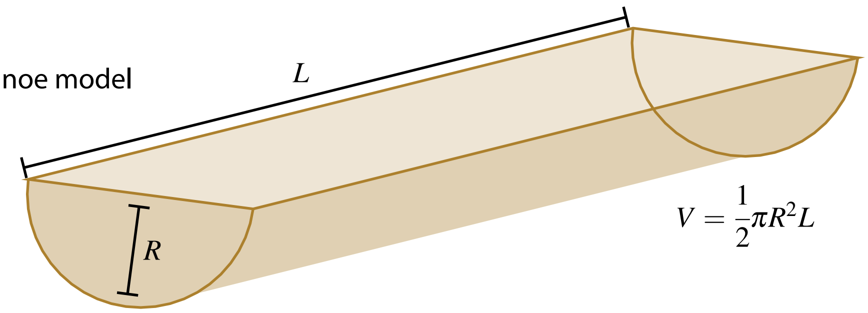
c. Let's figure out the depth and width later. For now, let's make the depth x meters. We don't want our canoe to be too wide or too narrow so we'll say that it's between x and $2x$ meters wide. Use your answer from Part 2b to fill out this table.

Dimension	**Measurement (meters)**
Length	At least 4.5
Width	Between x and $2x$
Depth	x

Part 3: Modeling the canoe

Now we need the volume of our canoe! We don't have an equation for this so we need to model the canoe with another shape that we do have an equation to. Let's use half a cylinder. If a cylinder has a length of L and radius of R, then its volume is $\pi R^2 L$. So the volume for half of this cylinder is $V = \frac{1}{2}\pi R^2 L$.

Simplified canoe model



a. What does the volume equation become when we replace R and L with an appropriate depth and length?
Hints:
Do we know what x is?
Do we need to use the *minimum* length?

> We can replace R with x and L with 4.5 or greater. Thus we get $V = \frac{1}{2}\pi x^2(4.5)$ or equivalently $V = 2.25\pi x^2$. Note that we can get many more answers by using longer L values. For example, $V = \frac{1}{2}\pi x^2(5)$ is also okay, but $V = \frac{1}{2}\pi x^2(4)$ isn't.

b. Of course, the shape of a canoe is not exactly a cylinder, but it is pretty close. What are some other shapes that you have the volume equations for? Do these shapes make better or worst models than the cylinder?

> Shapes like cubes and spheres make pretty poor canoe models. Triangular prisms can be made into pretty good models.

c. If you think that you have a better model than half a cylinder, you may use that instead.

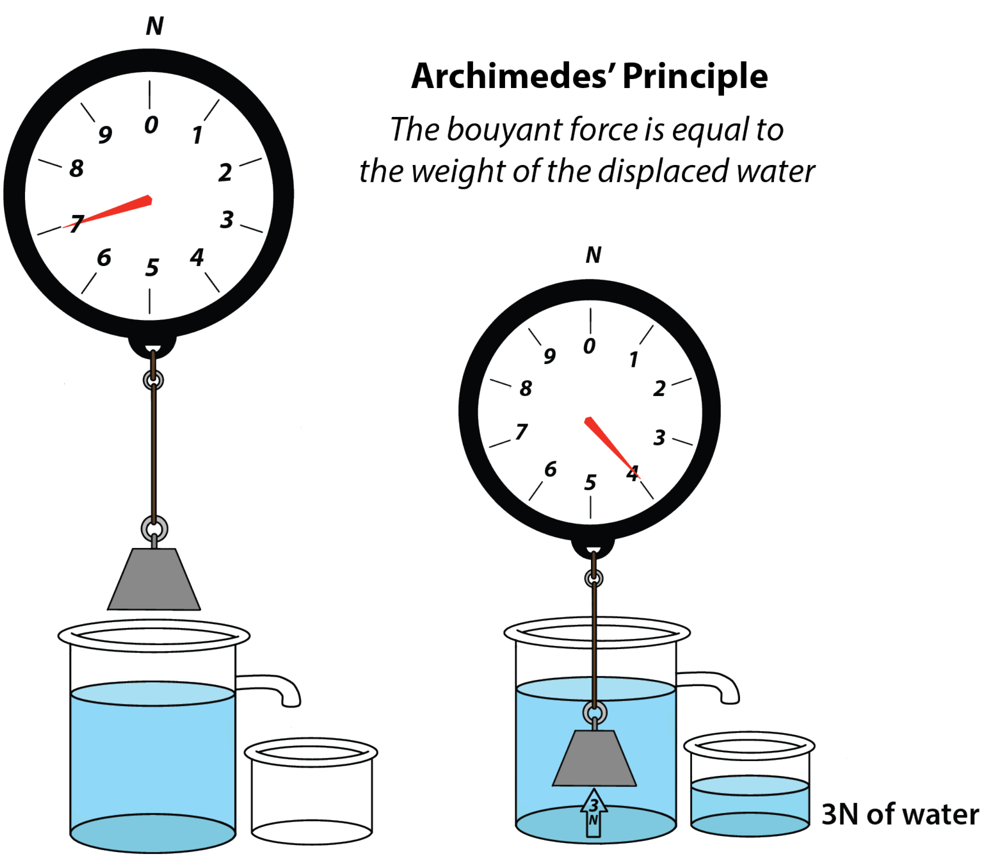
Part 4: Buoyant force and Archimedes' Principle

Now we have to make sure that our canoe floats. To do this, we have to learn about buoyancy and Archimedes' Principle.

a. Do some research on the Internet about Archimedes' Principle. What did you learn? What are some things about Archimedes' Principle that are still confusing? Share with your class and talk about it together.

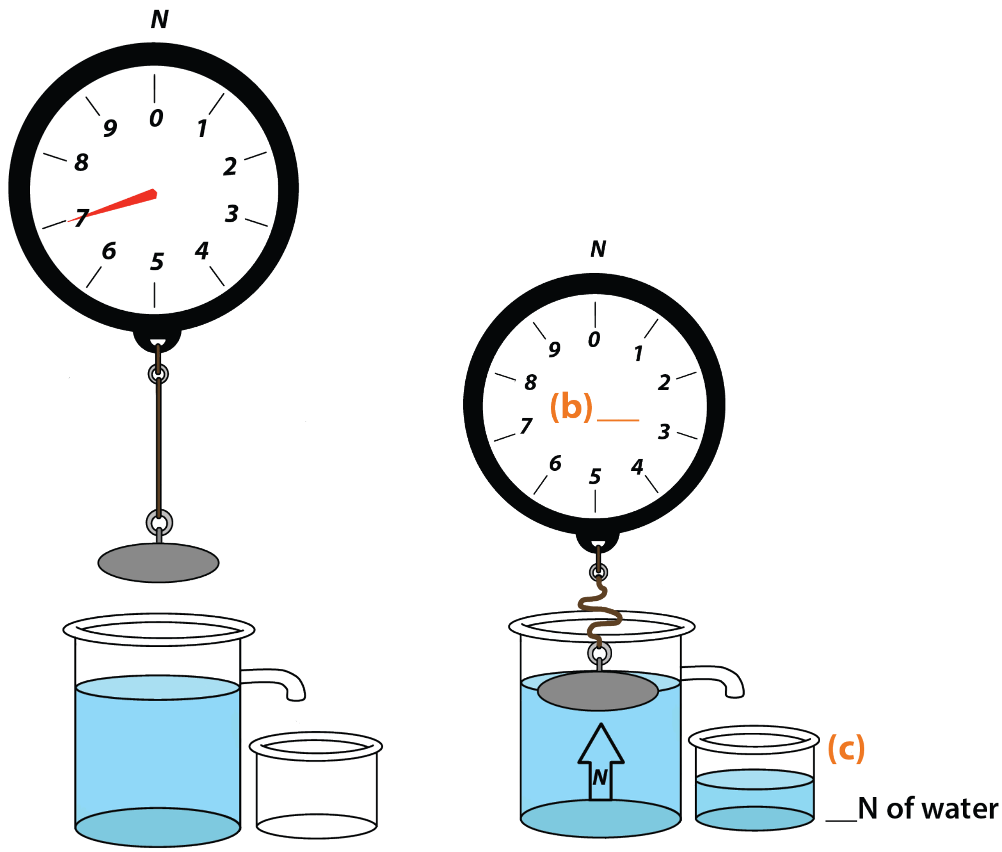
Answers will vary between students. Archimedes' Principle is described below.

Take a look at the first example in the picture. We have an object that weighs 7 N in the air. When we drop the object into the water, some of the water is displaced (moved out of the way) and spills out. The weight of the displaced water is 3 N and the object now weighs only 4 N. **Weight is a force that pushes down and buoyancy is a force that pushes up**, and Archimedes' Principle says that the buoyant force is equal to the weight of the displaced water.



b. Look at the second example. This time, the object floats when lowered into the water. What is the weight of the object when it is lowered into the water?

Since it floats, the object has a weight of 0 Newton.



c. Although the object floats, some water is still displaced. What is the weight of the displaced water?

The object used to weigh 7 N. To weigh 0 N now, it must displace 7 N of water as buoyant force (7 N $-$ 7 N $=$ 0 N).

d. Make sure that your answers to Parts 3b and 3c make sense when you compare it to the example. Discuss with the class to make sure that everyone is on board.

e. Notice that the object is not completely submerged. So it can actually displace more water if we push it down. If you can finish this sentence, then you are ready to move on.
The weight of the water displaced by an object that can **float** is _____ than the weight of the object.

○ less than
● equal or greater than

Part 5: Final Design

a. How many Newtons of water does the canoe have to displace in order to float? Look back at your answers from Part 1 if you're not sure.

> The canoe needs to displace $(483 \times g)$ N (or about 4733.4 N, if you're using $g \approx 9.8\text{m}/\text{s}^2$).

b. Density is defined as Mass ÷ Volume. So if we have a volume of water, and we multiply it by the density of water, then we get the mass of water. If we multiply that by the acceleration due to gravity, then we have the weight of water. Find the weight of the water that the canoe displaces. *Hints:*

1. You have a formula for the volume in Part 3.
2. The density of water is 1000kg/m^3.
3. The acceleration due to gravity is g m/s^2.
4. Your final answer should be an expression with the variable x (from your volume formula).

> Answers will vary. If we're using the half cylinder of minimum length, then our volume is $2.25\pi x^2$. So our mass is $(2.25\pi x^2) \times 1000$ and our weight is $(2.25\pi x^2) \times 1000 \times g = 2250\pi g x^2$. If we're using the decimal approximation of $g \approx 9.8\text{m}/\text{s}^2$ then we have a weight of $22050\pi x^2$ Newtons.
>
> Note that the students may be using a different model.

c. At what x will the canoe be deep enough to float with all the paddlers inside?
Hint: Use your last two answers.

> We need to set the weight of the displaced water to be equal or greater than the weight of the canoe. Then we solve for x.
>
> $$\begin{aligned} \text{weight of displaced water} &\geq \text{weight of canoe} \\ 2250\pi g x^2 &\geq (483 \times g) \\ (2250\pi g)x^2 &\geq 483g \\ \div(2250\pi g) & \quad\;\; \div(2250\pi g) \\ x^2 &\geq \frac{483g}{2250\pi g} \\ x^2 &\geq \frac{483}{2250\pi} \\ x^2 &\geq 0.215 \end{aligned}$$
>
> This gives us the solution $x \geq 0.463$ and the impossible solution $x \leq -0.463$. So we can say that x has to be at least 0.463 meters.
>
> You may also let the students use `www.desmos.com` to solve this.

d. Did you notice that the depth x is pretty small? It's actually not very safe to have such shallow canoe. Choose a larger x that you think adults would feel comfortable in. Now that you have x, you can use the table in Part 2 to help you choose your measurements.

We have sample answers below. However, we should note that the answers will vary and how much they vary is explained a table in Part 2.

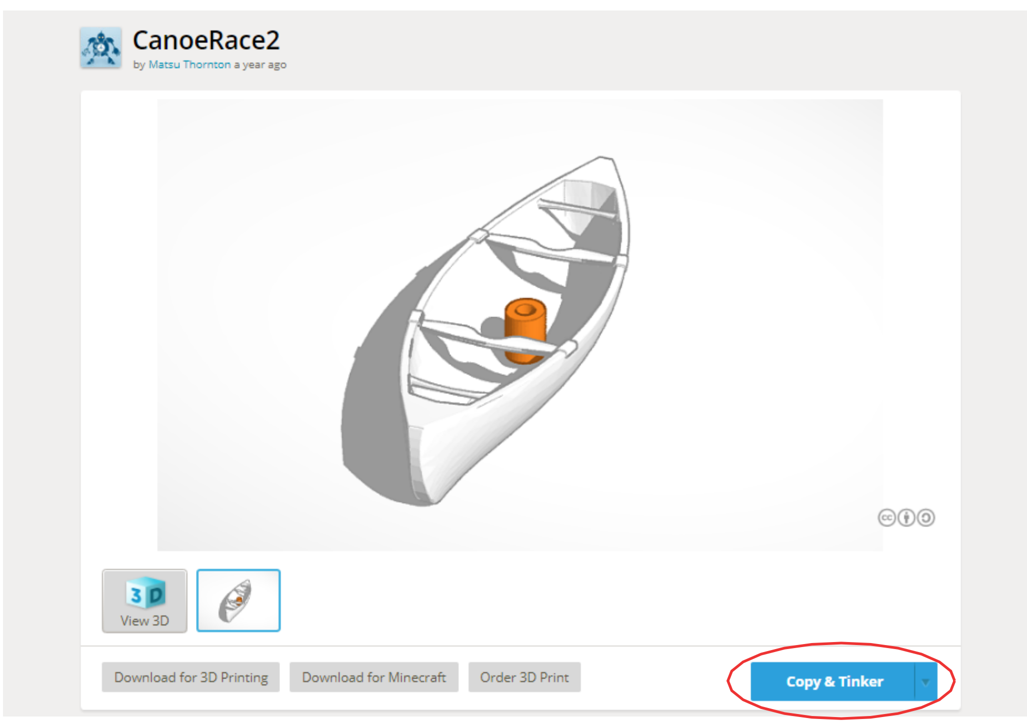
Dimension	Measurement (meters)
Real length	5
Real width	0.75
Real depth	0.5

e. Now let's shrink the canoe down for the 3D printer. Dilate all your dimensions by a scale factor of 1/50. You also have to change to millimeters.

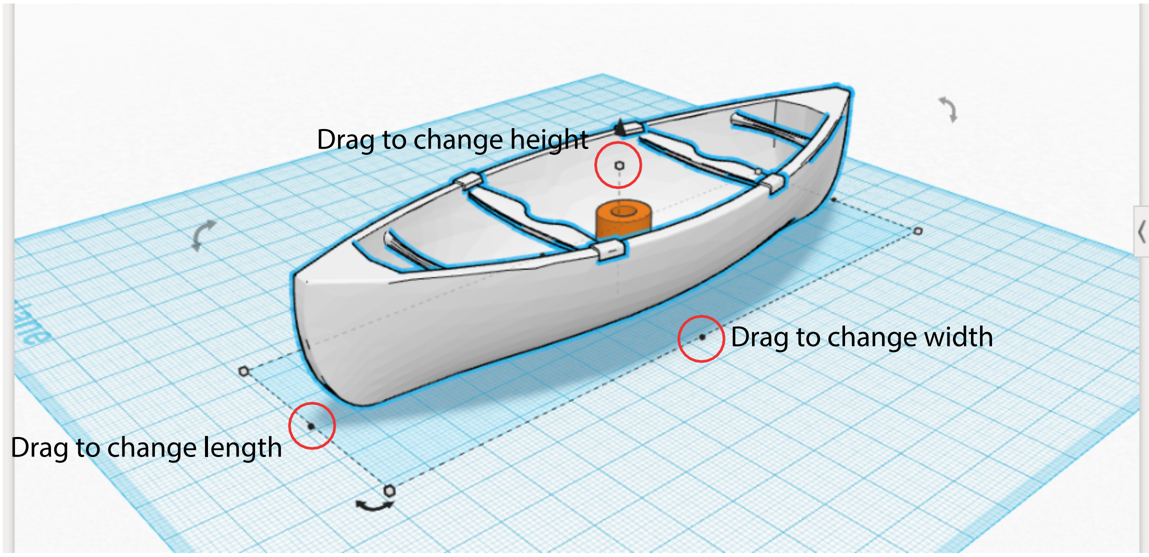
Dimension	3D Model measurement (millimeters)
Model length	100
Model width	15
Model depth	10

Part 6: Printing

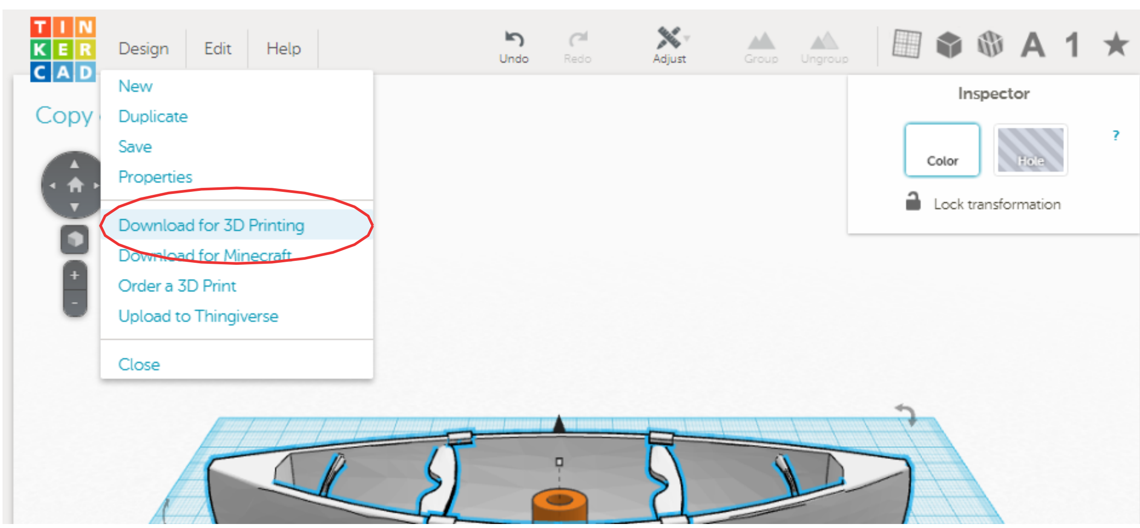
a. Open the canoe model at `https://tinkercad.com/things/9LTGjEvN0LJ`. Log in or register for a free account and press the "Copy & Tinker" button shown in figure below.



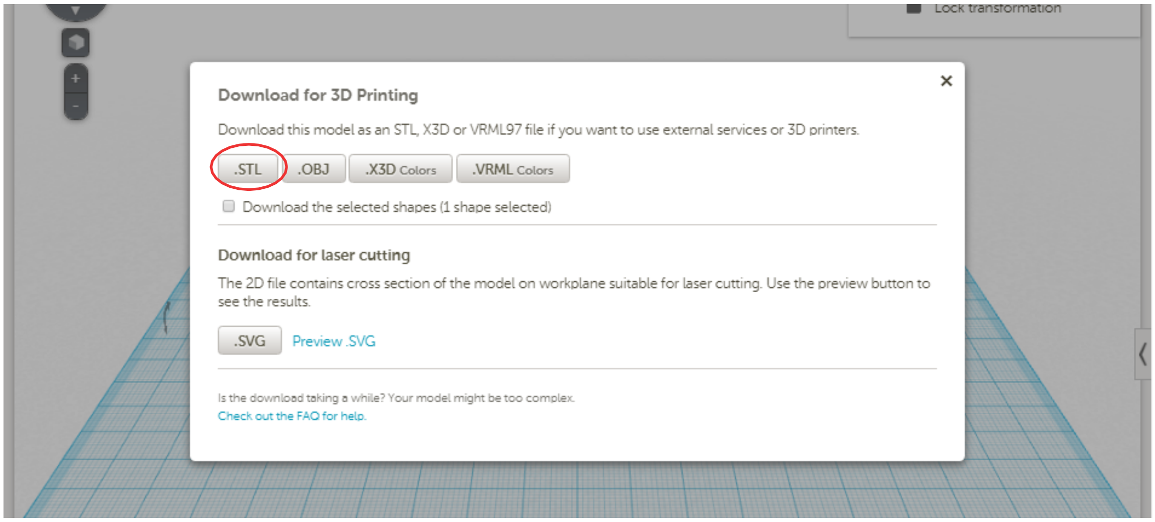
b. Now we will make the canoe into the correct size. To adjust the length, width, and depth, use the indicated anchor points in the next figures.



c. After you complete the canoe shape, go to the "design" menu and select download for 3D printing.



And choose .stl as your file type.



This will download a .stl file to your local hard drive depending on your download settings. Move the file to a folder where you know you can find it.

Rename the file using the following format:
<SchoolName>_<Class>_<length>_<width>_<depth>.stl

Example: WaipahuInt_MATH101_125_24_20.stl

d. Work with your school to get your model 3D printed.

For schools that are partnered with Ne'epapa Ka Hana, students can upload their file(s) to this dropbox for 3D printing.
`https://www.dropbox.com/request/swbYbb7P2i0yC6zBH3wz`

We'll send the printed canoes and a link to watch the canoes being 3D printed!

e. When it is printed and delivered, please sand and paint your canoe!